

# The Lorentz-violating extension of the Standard Model

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**Abstract.** Quantum-gravity effects are expected to be suppressed by the Planck mass. For experimental progress it is therefore important to identify potential signatures from Planck-scale physics that are amenable to ultrahigh-precision tests. It is argued that minuscule violations of Lorentz and CPT symmetry are candidate signals. In addition, theoretical and experimental aspects of the Standard-Model Extension, which describes the emergent low-energy effects, are discussed.

## 1 Motivation

An important open question in our understanding of nature at its fundamental level concerns a unified quantum description of all fundamental interactions including gravity. Such a theory is likely to become dominant only as the Planck scale is approached, so that quantum-gravity effects are expected to be minuscule at presently attainable energies. Moreover, the absence of a fully realistic and viable candidate underlying theory provides a major obstacle for the identification of concrete quantum-gravity signatures that can be searched for in present-day or near-future experiments.

A possible approach to overcome this phenomenological issue is to determine exact relations in the currently accepted laws of physics that may be violated in prospective fundamental theories and that can be tested with ultrahigh-precision. Symmetries typically satisfy these criteria. For example, Lorentz and CPT invariance are cornerstones of our present understanding of nature at the fundamental level, and a variety of Lorentz and CPT tests belong to the most sensitive null experiments available. Lorentz and CPT violation is also a key feature of certain approaches to underlying physics.

For example, couplings varying on cosmological scales are one possible source of Lorentz and CPT violation [1]. This fact does not come as a surprise: parameters dependent on spacetime break translational invariance, and translations, rotations, and boosts are linked in the Poincaré group. Thus, violations of translation symmetry can also affect Lorentz invariance. This can be understood intuitively as follows. The equations of motion typically contain the gradient of the coupling, which selects a preferred direction in spacetime leading to apparent Lorentz violation.

In this talk, we discuss some theoretical and experimental aspects of the Standard-Model Extension (SME) [2–7], which is the low-energy framework for Lorentz-breaking effects. The SME is a dynamical model constructed to contain all Lorentz- and CPT-violating lagrangian terms consistent with coordinate

independence, which is a fundamental requirement to be discussed below. To date, numerous Lorentz- and CPT-violation tests involving hadrons [8–21], protons and neutrons [22–31], electrons [31–41], photons [42–47], muons [48–50], and neutrinos [2,51–54] have been analysed or identified within the framework of the SME.

The outline of this talk is as follows. In Sec. 2, we analyse the requirement of coordinate independence and its implementation. Section 3 contains a discussion of the SME. In particular, its construction is reviewed, its generality is addressed, and its advantages are summarized. In Sec. 4, varying couplings are investigated from the perspective of providing a potential source of Lorentz and CPT violation. Section 5 comments on kinematical Lorentz tests with modified dispersion relations. The conclusions are contained in Sec. 6.

## 2 Coordinate independence

One of the most fundamental principles in physics is coordinate independence. The need for this principle in the presence of Lorentz breaking is well established [2,3,55], and it has served as the basis for the construction of the SME. However, in some investigations of Lorentz and CPT violation coordinate-*dependent* physics emerges, and occasionally Lorentz-symmetry breakdown is identified with the loss of coordinate independence. For these reasons, it is appropriate to review this fundamental principle and its implementation. It then also becomes clear how coordinate independence provides a rough classification of different approaches to Lorentz and CPT breaking.

A certain choice of labeling events in space and time is called a coordinate system. Such a labeling scheme is typically observer dependent and thus arbitrary to a large extent. Coordinate systems belong to the most common and important *tools* for the description of processes occurring in nature, but they fail to possess *physical reality*: the choice of coordinates must leave the physics unaffected. This principle of coordinate independence is fundamental in science. It assures that the physics remains independent of the observer, and it is therefore also called observer invariance. Coordinate independence is guaranteed when spacetime is given a manifold structure and physical quantities are represented by geometric objects, such as tensors or spinors.

Coordinate-*dependent* physics does break Lorentz symmetry. However, the converse (i.e., Lorentz violation is associated with the loss of coordinate independence) is a common misconception. The principle of coordinate invariance is, in fact, independent from Lorentz symmetry. For instance, Newton’s law of gravitation and nonrelativistic classical mechanics are non-invariant under Lorentz transformations but can be formulated in the coordinate-free language of 3-vectors. The Lorentz transformations acquire a significant role only on lorentzian spacetime manifolds where they implement changes between local inertial frames.

Even on a lorentzian manifold, Lorentz symmetry may be broken. This fact can be illustrated in the conventional context of a classical point particle of mass

$m$  and charge  $q$  subjected to an external electromagnetic field  $F^{\mu\nu}$ . The equation of motion for such a particle reads

$$m \frac{dv^\mu}{d\tau} = q F^{\mu\nu} v_\nu, \quad (1)$$

where  $\tau$  is the particle's proper time and  $v^\mu$  is its 4-velocity. Equation (1) remains valid in all coordinate systems because it is a tensor equation. Thus, observer Lorentz symmetry is maintained. However, the external  $F^{\mu\nu}$  background violates, for example, symmetry under arbitrary rotations of the charge's trajectory. Among the consequences of this noninvariance is the violation of angular-momentum conservation for the particle. Note the difference to coordinate changes, which leave unaffected the physics: here, only the trajectory is rotated, so that its orientation with respect to  $F^{\mu\nu}$  can change. One then says that particle Lorentz symmetry is violated, despite the presence of observer invariance [2,55]. It is important to point out that in the above example, the background  $F^{\mu\nu}$  is a local electromagnetic field caused by other 4-currents that can in principle be controlled. Our external-field illustration therefore fails to contain Lorentz violation as a fundamental property of an effective vacuum.

The above discussion suggests the possibility of classifying different approaches to Lorentz violation by their behavior under coordinate changes. In the remaining part of this section, we discuss such a classification.

**Models with coordinate-dependent physics.** Although it appears to be impossible to perform meaningful scientific investigations involving coordinate-dependent physics, such approaches to Lorentz breaking have been considered in the literature. More specifically, there have been two suggestions in the context of neutrino phenomenology: the first one forces the masses of particle and antiparticle to be different [56], while the second one attempts to build a model from positive-energy eigenspinors only [57]. Both approaches are known to involve coordinate-dependent off-shell physics [58,59]. In what follows, we do not consider these approaches further.

**Coordinate-covariant models involving non-lorentzian manifolds.** Another possibility to speculate how Lorentz invariance might be lost is the following. Local inertial frames have a structure different from the usual Minkowskian one, so that Lorentz transformations no longer implement changes between inertial coordinates, i.e., observer Lorentz invariance is *replaced* by observer invariance under some other symmetry transformation. But nevertheless, coordinate independence is maintained. This point of view is taken in the so called “doubly special relativities” [60,61]. We mention that both the viability and the physical interpretation of this approach appear to be controversial at the present time [62–66]. We leave such Lorentz-symmetry *deformations* unaddressed in the present work.

**Coordinate-independent models involving nontrivial vacua.** In this approach, a fully Lorentz-covariant underlying model generates a tensorial background resulting in apparent Lorentz violation. The basic idea parallels that of our above external-field example. However, the background is outside of experimental control and must be viewed as a property of the effective vacuum. Because

of the lorentzian structure of the underlying manifold and the usual Lorentz-covariant dynamics at the fundamental level, this approach appears closest to established theories. The physical effects in such models are perhaps comparable to those inside certain crystals: the physics remains independent of the chosen coordinates, but particle propagation, for example, can be direction dependent. As an immediate consequence, one can locally still work with the metric  $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ , particle 4-momenta are still additive and still transform in the usual way under coordinate changes, and the conventional tensors and spinors still represent physical quantities.

Such nontrivial effective vacua can arise in various theories beyond the Standard Model. For instance, the possibility of spontaneous Lorentz and CPT breaking in the framework of string field theory was discovered more than a decade ago [67–74]. Subsequent studies have considered other mechanisms for Lorentz-violating vacua including spacetime foam [75,76], nontrivial spacetime topology [77], loop quantum gravity [78,79], realistic noncommutative field theories [80–83], and spacetime-varying couplings [1,84].

### 3 The Standard-Model Extension

The next step after determining general low-energy features is the identification of specific experimental signatures for Lorentz breaking and the theoretical analysis of Lorentz-violation searches. This task is most conveniently performed within a suitable test model. Many Lorentz tests are motivated and analysed in purely kinematical frameworks allowing for small violations of Lorentz symmetry. Examples are Robertson’s framework, its Mansouri-Sexl extension, the  $c^2$  model, and phenomenologically constructed modified dispersion relations. But it is also clear that the implementation of general dynamical features significantly increases the scope of Lorentz tests. For this reason, the SME mentioned in the introduction has been developed. However, the use of dynamics in Lorentz-violation searches has recently been questioned on the grounds of framework dependence. We disagree with this claim and begin with a few arguments in favor of a dynamical test model.

Such a model is constrained by the requirement that known physics must be recovered in certain limits, despite some freedom in introducing dynamical features compatible with a given set of kinematics rules. In addition, it seems difficult and may even be impossible to construct an effective theory containing the Standard Model with dynamics significantly different from that of the SME. We also mention that kinematical analyses are limited to only a subset of potential Lorentz-violating signatures from fundamental physics. From this viewpoint, it is desirable to explicitly implement dynamical features of sufficient generality into test models for Lorentz and CPT symmetry.

**The generality of the SME.** In order to understand the generality of the SME, we review the main elements of its construction [2,3]. Starting from the usual Standard-Model lagrangian  $\mathcal{L}_{\text{SM}}$  one adds Lorentz-violating modifications

$\delta\mathcal{L}$ :

$$\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L} . \quad (2)$$

Here, the SME lagrangian is denoted by  $\mathcal{L}_{\text{SME}}$ . The correction  $\delta\mathcal{L}$  is formed by contracting Standard-Model field operators of any dimensionality with Lorentz-breaking tensorial coefficients that describe the nontrivial vacuum discussed in the previous section. To ensure coordinate independence, this contraction must yield observer Lorentz scalars. It becomes thus apparent that all possible contributions to  $\delta\mathcal{L}$  determine the most general effective dynamical description of Lorentz violation at the level of observer Lorentz-invariant quantum field theory.

Potential Planck-scale features, such as non-pointlike elementary particles or a discretized spacetime, are unlikely to invalidate the above effective-field-theory approach at presently attainable energies. On the contrary, the phenomenologically successful Standard Model is normally understood as an effective-field-theory approximation for more fundamental physics. If fundamental physics indeed exhibits minuscule Lorentz-breaking effects, it would seem contrived to consider low-energy effective models outside the framework of quantum field theory. We finally mention that the need for a low-energy description beyond effective field theory is also unlikely to arise in the context of candidate underlying models with novel Lorentz-invariant aspects, such as additional particles, new symmetries, or large extra dimensions. Lorentz-symmetric modifications can therefore be implemented into the SME, if necessary [85–87].

**Advantages of the SME.** The SME permits the identification and direct comparison of essentially all currently feasible experiments searching for Lorentz and CPT violation. In addition, certain limits of the SME correspond to classical kinematics test models of relativity (such as the aforementioned Robertson’s framework, its Mansouri-Sexl extension, or the  $c^2$  model) [44]. Another advantage of the SME is the possibility of implementing additional desirable conditions besides coordinate independence. For instance, one can choose to require translational invariance,  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  gauge symmetry, power-counting renormalizability, hermiticity, and pointlike interactions. These conditions further restrict the parameter space for Lorentz breaking. One can also adopt simplifying choices, such as rotational invariance in certain coordinate systems. This latter assumption together with additional simplifications of the SME has been discussed in Ref. [51].

## 4 Varying couplings and the SME

In this section, we construct a classical cosmological solution in the framework of the pure  $N = 4$  supergravity in a four-dimensional spacetime. We show that this solution leads to a spacetime variation of both the fine-structure parameter  $\alpha$  and the electromagnetic  $\theta$  angle. Such a model fails to be fully realistic in detail, but it is a limit of the  $N = 1$  supergravity in 11 dimensions, which is contained in M-theory. It could therefore give insight into generic features expected to arise in a promising candidate underlying theory. We illustrate the associated

Lorentz-violating effects by looking at the  $\theta$ -angle variation, which gives rise to the  $(k_{AF})^\mu$  term contained in the SME.

**Basics of the model.** In Planck units, the bosonic lagrangian for our  $N = 4$  supergravity in four dimensions takes the following form [88]:

$$\mathcal{L} = \sqrt{g} \left( -\frac{1}{2}R - \frac{1}{4}MF_{\mu\nu}F^{\mu\nu} - \frac{1}{4}NF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B}{4B^2} \right). \quad (3)$$

Here,  $g_{\mu\nu}$  represents the metric, and we have assumed that only one graviphoton,  $F_{\mu\nu}$ , is excited. The dual  $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$  is defined as usual. The model also contains two scalars  $A$  and  $B$  that can be identified with an axion and a dilaton. The dependence of the couplings  $M$  and  $N$  on the scalars is fixed by the supergravity framework [1]:

$$M = \frac{B(A^2 + B^2 + 1)}{(1 + A^2 + B^2)^2 - 4A^2}, \quad N = \frac{A(A^2 + B^2 - 1)}{(1 + A^2 + B^2)^2 - 4A^2}. \quad (4)$$

In the pure  $N = 4$  supergravity in four dimensions, the graviphoton couples nonminimally to matter. Although the internal  $\text{SO}(4)$  symmetry can be gauged [89], we adopt a phenomenological approach: in a realistic situation, the vector-matter interaction must be minimal. In what follows, we therefore can identify the graviphoton  $F_{\mu\nu}$  with the electromagnetic field.

**Supergravity cosmology.** Next, we consider the above model in a cosmological context. We begin with the standard assumption of a homogeneous and isotropic universe. This implies that  $F_{\mu\nu} \simeq 0$  on cosmological scales. We further take the universe to be flat, i.e.,  $k = 0$ . This is justified in light of recent measurements [90–92]. The Friedmann-Robertson-Walker (FRW) line element has the conventional form:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2). \quad (5)$$

Here, the usual comoving coordinates have been adopted and  $a(t)$  denotes the cosmological scale factor. As a consequence of the above assumptions, not only the scale factor, but also  $A$  and  $B$  depend on  $t$  only.

For phenomenological reasons it is necessary to model the known matter content of the universe. We employ a standard approach and incorporate the energy-momentum tensor of dust,  $T_{\mu\nu}$ , into our framework. If  $u^\mu$  is the unit timelike vector orthogonal to the spatial hypersurfaces and  $\rho(t)$  is the energy density of the dust, the usual arguments imply that  $T_{\mu\nu} = \rho u_\mu u_\nu$ . In the present model, this type of matter arises from the fermionic sector of our supergravity framework. At tree level, the scalars  $A$  and  $B$  do not couple to the fermions [88], so that we can take  $T_{\mu\nu}$  as conserved separately.

It turns out that the equations of motion for our supergravity cosmology can be integrated analytically. For example, the time dependences of  $A$  and  $B$  are given by [1]

$$A = \pm \lambda \tanh \left( \frac{1}{\tau} - \frac{1}{\tau_0} \right) + A_0, \quad B = \lambda \operatorname{sech} \left( \frac{1}{\tau} - \frac{1}{\tau_0} \right), \quad (6)$$

where  $\lambda$ ,  $1/\tau_0$ , and  $A_0$  are integration constants. The parameter time  $\tau$  is defined by  $\tau = \sqrt{3}/4 \operatorname{arccoth}(\sqrt{3c_n/4c_1} t + 1)$ , which contains two more integration constants  $c_n$  and  $c_1$ . The solution (6) implies that both  $A$  and  $B$  approach constant values at late times  $t \rightarrow \infty$ . Thus, the values of the axion  $A$  and the dilaton  $B$  become fixed in our supergravity cosmology, despite the absence of a dilaton potential. This is essentially a consequence of energy conservation.

**Varying couplings.** Next, we consider excitations of  $F_{\mu\nu}$  in the axion-dilaton background determined by Eq. (6). From a phenomenological viewpoint, we can take these excitations to be localized in spacetime regions small on cosmological scales. It is therefore appropriate to work in local inertial frames.

The conventional electrodynamics lagrangian in inertial coordinates can be taken as

$$\mathcal{L}_{\text{em}} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{\theta}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (7)$$

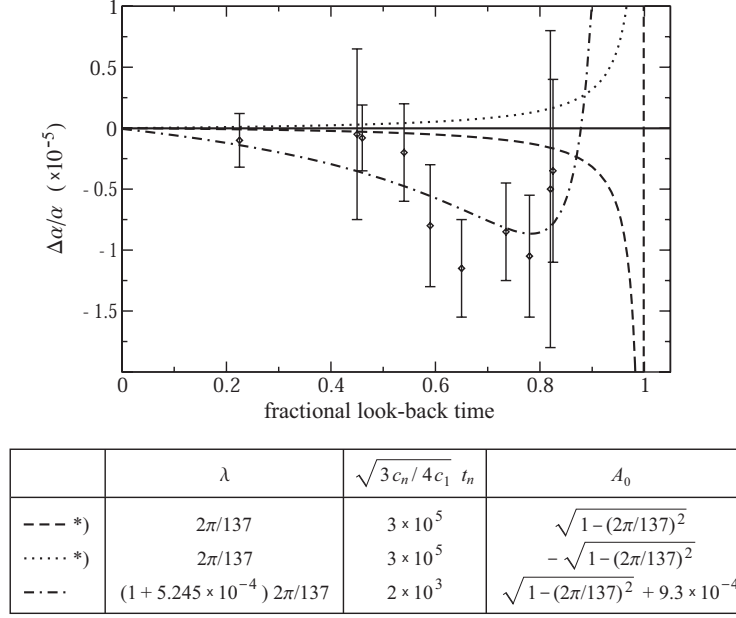
where we have allowed for a nontrivial  $\theta$ -angle. The electromagnetic coupling is denoted by  $e$ . Comparison with our supergravity lagrangian (3) shows that we can identify  $e^2 \equiv 1/M$  and  $\theta \equiv 4\pi^2 N$ . It is important to note that  $M$  and  $N$  are determined by the axion-dilaton background (6), so that  $e$  and  $\theta$  become functions of the comoving time  $t$ . In arbitrary local inertial frames, the electromagnetic coupling and the  $\theta$  angle therefore exhibit related spacetime dependences.

We remark in passing that the functional dependence of the fine-structure parameter  $\alpha = e^2/4\pi$  on the comoving time can vary qualitatively with the choice of model parameters. Figure (1) displays relative variations  $\Delta\alpha/\alpha$  of the fine-structure parameter versus fractional look-back time  $1-t/t_n$  to the big bang. Here,  $t_n$  denotes the present age of the universe and  $\Delta\alpha = \alpha(t) - \alpha(t_n)$ . We have set the parameter  $1/\tau_0$  to zero. The solid line corresponds to a constant  $\alpha$ . Each broken line represents a set of nontrivial choices for  $\lambda$ ,  $\sqrt{3c_n/4c_1}t_n$ , and  $A_0$ . Input parameters leading to a variation consistent with the Oklo constraints [95–97] are labeled with an asterisk. The qualitative differences in the various plots, the nonlinear features, and the sign change for  $\dot{\alpha}$  in the two cases with positive  $A_0$  are apparent. Figure (1) also depicts the recent experimental results [94] obtained from measurements of high-redshift spectra over periods of approximately  $0.6t_n$  to  $0.8t_n$  assuming  $H_0 = 65 \text{ km/s/Mpc}$  and  $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ .

**Lorentz violation.** The Lorentz violation associated with varying couplings becomes perhaps most transparent in the equations of motion. Incorporating charged matter described by a 4-current  $j^\nu$ , lagrangian (3) yields in a local inertial frame:

$$\frac{1}{e^2}\partial_\mu F^{\mu\nu} - \frac{2}{e^3}(\partial_\mu e)F^{\mu\nu} + \frac{1}{4\pi^2}(\partial_\mu \theta)\tilde{F}^{\mu\nu} = j^\nu. \quad (8)$$

Note that in the limit of spacetime-constant  $e$  and  $\theta$ , the conventional inhomogeneous Maxwell equations are recovered. In the axion-dilaton background (6), however, the terms containing the gradients of  $e$  and  $\theta$  are associated with apparent Lorentz violation: since the gradients can be treated as effectively non-dynamical and constant on small cosmological scales, they select a preferred



**Fig. 1.** Relative change in the fine-structure parameter versus fractional look-back time to the big bang for various choices of integration constants [93]. Note the qualitative differences and nonlinear features. Also shown are the Webb data [94].

direction in the local inertial frame. As a result, symmetry under boosts and rotations of electromagnetic fields is broken. Note that this type of Lorentz violation is not a feature of the particular coordinate system chosen. If a gradient is nonzero in one local inertial frame associated with a small spacetime region, it is nonzero in *all* local inertial frames associated with the region in question.

By contrast, such a Lorentz-symmetry breakdown is absent in conventional FRW cosmologies that fail to contain spacetime-dependent scalars. Although global Lorentz invariance is usually violated, local Lorentz-symmetric inertial frames always exist. It is also important to note that the above source for Lorentz-violating effects is not a unique feature of our supergravity cosmology. Equation (8) illustrates that any smooth spacetime dependence of the couplings  $e$  and  $\theta$  on cosmological scales leads to such effects. It is theoretically attractive to associate varying couplings with quantum scalar fields acquiring spacetime-dependent expectation values. However, from the perspective of Lorentz violation, classical scalars can be employed equally well. In fact, the variation of the coupling need not necessarily be driven by dynamical fields at all. This suggests that the above type of Lorentz breaking is a common feature of any model with spacetime-varying couplings.

Next, we study how the effects of this mechanism fit into the framework of the SME and how our supergravity model helps to resolve conceptual issues in



quantum field theories incorporating Lorentz breaking. This is best illustrated by considering the  $\theta$ -angle term. An integration by parts yields an equivalent form of the electrodynamics lagrangian (7) in a local inertial frame:

$$\mathcal{L}'_{\text{em}} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi^2}(\partial_\mu\theta)A_\nu\tilde{F}^{\mu\nu}. \quad (9)$$

The the last term on right-hand side of Eq. (9) gives a Chern-Simons-type contribution to the action. Such a term is contained in the minimal SME, and we can identify  $(k_{AF})_\mu \equiv e^2\partial_\mu\theta/8\pi^2$ . The presence of a nonzero  $(k_{AF})_\mu$  in Eq. (9) demonstrates explicitly Lorentz and CPT breaking at the lagrangian level.

The case of constant  $e$  and  $(k_{AF})_\mu$  has been discussed extensively in the literature [42,3,4,77,98]. Then, lagrangian (9) becomes translationally invariant resulting in an associated conserved energy that fails to be positive definite. This usually leads to instabilities in the theory, and the question arises how this problem is avoided in the present context of our positive-definite supergravity model.<sup>1</sup>

Although in most models a Chern-Simons-type term is assumed to arise in a fundamental theory,  $(k_{AF})_\mu$  is typically treated as constant and nondynamical at low energies. In our supergravity cosmology, however,  $(k_{AF})_\mu$  is associated with the dynamical scalars  $A$  and  $B$ . Excitations of  $F_{\mu\nu}$  therefore result in perturbations  $\delta A$  and  $\delta B$  in the axion-dilaton background (6). As a consequence, the energy-momentum tensor  $(T^{\text{b}})^{\mu\nu}$  of the background acquires an additional contribution,  $(T^{\text{b}})^{\mu\nu} \rightarrow (T^{\text{b}})^{\mu\nu} + \delta(T^{\text{b}})^{\mu\nu}$ . One can show [1] that the contribution  $\delta(T^{\text{b}})^{\mu\nu}$  does indeed compensate for the negative energies associated with a nonzero  $(k_{AF})_\mu$ .

## 5 Threshold analyses

Within the SME, it is straightforward to verify that Lorentz violation typically modifies one-particle dispersion relations [42,2,3,5]. This feature permits the prediction of possible experimental signatures for Lorentz-symmetry breakdown based purely on kinematical arguments. For instance, primary ultrahigh-energy cosmic rays (UHECR) at energies up to eight orders of magnitude below the Planck scale have been observed. At such energies, Lorentz-violating effects might be pronounced relative to the ones in low-energy experiments. This leads to potentially observable threshold modifications for particle reactions, an idea that has been adopted in many recent studies of Lorentz-symmetry breakdown [51,99–103]. However, it is known [55] that some threshold analyses employ phenomenologically constructed dispersion relations that violate physics principles more fundamental than Lorentz symmetry.

In this section, we investigate how some of the arbitrariness in the construction of dispersion-relation modifications can be removed. Our study relies

<sup>1</sup> The conserved symmetric energy-momentum tensor for the lagrangian (3) acquires no contribution from the  $N$  term because the latter is independent of  $g^{\mu\nu}$ . The other terms contributing to the energy density are positive definite.

on the principle of coordinate invariance and on the condition of compatibility with an effective dynamical framework like the SME. These two features appear fundamental enough for being physical requirements, while maintaining relative independence of the details of the Planck-scale theory. We also discuss causality and positivity, features that further add to the viability of threshold analyses. Throughout we assume exact conservation of energy and momentum.

**Coordinate-independent dispersion relations.** As argued in Sec. 2, coordinate independence is essential in physics, despite the presence of Lorentz violation. In the published literature, the usual ansatz for modified dispersion relations is of the following form:

$$E^2 - \mathbf{p}^2 = m^2 + \delta f(E, \mathbf{p}) , \quad (10)$$

where  $m$  is the usual mass parameter and  $p^\mu = (E, \mathbf{p})$  the 4-momentum. The function  $\delta f(E, \mathbf{p})$  controls the extent of the Lorentz violation. Coordinate independence requires  $\delta f$  to be a scalar, so that

$$\delta f(E, \mathbf{p}) = \sum_{n \geq 1} T_{(n)}^{\overbrace{\alpha\beta \cdots}^{n \text{ indices}}} \underbrace{p_\alpha p_\beta \cdots}_{n \text{ factors}} . \quad (11)$$

Here,  $T_{(n)}^{\alpha\beta \cdots}$  denotes a constant tensor of rank  $n$  representing the Lorentz-breaking background. The tensor indices  $\alpha, \beta, \dots$  are distinct but each one is contracted with a 4-momentum factor. This ensures that all terms in the sum are observer Lorentz invariant. Under mild assumptions, Eq. (11) determines the most general Lorentz-violating dispersion relation compatible with coordinate independence.

The implications of the general coordinate-independent ansatz (11) can be illustrated when the common assumption of rotation invariance in certain frames is made. In this case, the form of the Lorentz-violating tensor parameters  $T_{(n)}$  is constrained by the imposed rotational symmetry. As a consequence, the correction  $\delta f$  fails to contain odd powers of the 3-momentum magnitude  $|\mathbf{p}|$  [55]. This result is to be contrasted with the common occurrence of  $|\mathbf{p}|^3$  corrections in modified dispersion relations constructed by hand without reference to principles essential in physics.

Note that a correction  $\sim E|\mathbf{p}|^2$  is consistent with coordinate independence. Then, the question arises as to whether the usual ultrarelativistic relation  $E \simeq |\mathbf{p}|$  can introduce an effective  $|\mathbf{p}|^3$  modification. Although such a replacement may yield excellent approximations for the eigenenergies, it gives incorrect results in threshold analyses. This is intuitively reasonable because this replacement reintroduces the conventional degeneracy of the eigenenergies. An explicit example for the failure of the ultrarelativistic approximation is provided by photon decay into an electron-positron pair:

$$\gamma \rightarrow e^+ + e^- . \quad (12)$$

This process is kinematically forbidden in conventional physics. In the present Lorentz-violating context, the decay (12) is allowed when the coordinate-independent correction  $\sim E|\mathbf{p}|^2$  for photon, electron, and positron is used. However, when the correction term is approximated by  $|\mathbf{p}|^3$ , the process ceases to be kinematically permitted.

**Underlying dynamical framework.** The need for underlying dynamics can be illustrated with the following example. Consider the rotationally symmetric modified dispersion relation

$$(p^\mu p_\mu - m^2)^2 = |\mathbf{p}|^6. \quad (13)$$

Note that odd powers of  $|\mathbf{p}|$  are absent compatible with coordinate independence. After the usual reinterpretation of the negative-energy solutions, the particle and antiparticle energies are given by

$$E_{\pm}^{(\alpha)}(\mathbf{p}) = \sqrt{(-1)^\alpha \frac{|\mathbf{p}|^3}{M} + \mathbf{p}^2 + m^2}, \quad (14)$$

where  $\alpha = 1, 2$  labels the two possible particle (antiparticle) energies, which perhaps correspond to different spin-type states. An analysis of the photon decay (12) employing these eigenenergies reveals that six kinematically distinct decays have to be considered. Note, however, that angular-momentum conservation associated with the enforced rotational invariance may preclude some of the six reactions. A proper study of this case therefore requires dynamical concepts.

**Causality and positivity.** Causality and energy positivity are fundamental requirements in physics. However, one or both of these requirements can be violated in the presence of Lorentz violation [5]. From a conservative viewpoint, it is therefore natural to ask whether reaction-threshold kinematics is significantly affected when positivity and causality are imposed. Let  $M$  and  $m$  denote the scales of the fundamental theory and current low-energy physics, respectively. Then, the scale  $p_{s-c}$  for the occurrence of spacelike momenta (and thus negative energies in certain frames) or causality problems can be as low as [5]

$$p_{s-c} \sim \mathcal{O}(\sqrt{mM}). \quad (15)$$

For example, if  $m$  is the proton mass and  $M$  is taken to be the Planck scale, then  $p_{s-c} \sim 3 \times 10^{18}$  eV. UHECRs with a spectrum extending beyond  $10^{20}$  eV are often employed to bound Lorentz breaking or to suggest evidence for Lorentz violation. Thus, imposing causality and positivity could require modifications in threshold analyses.

As a specific example, consider the decay

$$\gamma \rightarrow \pi^0 + \gamma, \quad (16)$$

which is kinematically forbidden in conventional physics. The usual dispersion-relation modifications in the literature permitting this process are associated with causality or positivity violations: take the photon energy  $E$  to be given

by  $E = (\mathbf{p}^2 + \delta f(\mathbf{p}))^{1/2}$ , where  $\delta f(\mathbf{p})$  excludes a mass term<sup>2</sup> and depends only on the photon 3-momentum  $\mathbf{p}$ . Then, the point  $(E, \mathbf{p}) = (0, \mathbf{0})$  on the momentum-space lightcone must satisfy the modified dispersion relation. For some 3-momenta  $\mathbf{p} \neq \mathbf{0}$ , a nontrivial correction  $\delta f(\mathbf{p})$  forces  $E(\mathbf{p})$  to curve to the outside or inside of the lightcone leading to spacelike 4-momenta or superluminal group velocities, respectively. If, however, the photon dispersion relation is taken to be the conventional one, spacelike pions are required for the decay to occur.<sup>3</sup>

## 6 Conclusion

Although Lorentz and CPT invariance are deeply ingrained in the currently accepted laws of physics, there are a variety of candidate underlying theories admitting the violation of these symmetries. The ultrahigh sensitivity of many Lorentz and CPT tests therefore permits the experimental search for Planck-scale physics.

Spacetime-dependent couplings are one potential source for apparent Lorentz and CPT violation: the gradient of such couplings in the equations of motion selects a preferred spacetime direction in the effective vacuum. We have argued that variations of couplings are natural in a cosmological context of candidate fundamental theories.

The first-order Lorentz-violating effects resulting from varying couplings and other mechanisms for Lorentz-symmetry breakdown are described by the SME. At the level of effective quantum field theory, the SME is the most general dynamical framework for Lorentz and CPT violation that is compatible with the fundamental principle of coordinate independence.

Threshold analyses with modified dispersion relations are conceptually clean Lorentz tests and are best performed within the SME. Many purely kinematical threshold considerations in the literature are insufficient for bounding Lorentz breaking because they violate coordinate independence or other fundamental principles.

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<sup>2</sup> A mass term would yield a Lorentz-invariant (but gauge-symmetry violating) contribution, which is not of interest in the present context.

<sup>3</sup> The allowed phase space for the decay products in the case of a lightlike pion 4-momentum is a set of measure zero leading at best to a suppressed rate for the reaction.

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